AMERICAN NON-DIVIDEND PAYING CALL'S NON-EARLY EXERCISE

LARRY, ZHIRONG LI

Prove it is never optimal to exercise an American call on a non-dividend paying stock. (A put, though, is a different matter) .

The most direct approach is to use put-call parity. We know that

$$C_t - P_t = S_t - Ke^{-r(T-t)} \geq S_t - K$$

As a result,

$$C_t \geq (S_t - K)^+ .$$

The intrinsic value of an American call is $\max\{S_t - K, 0\} = (S_t - K)^+$ and the European call pricing formula is given by

$$C_t = S_t \mathcal{N}(d_1) - Ke^{-r(T-t)} \mathcal{N}(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

and

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Let D_t be the difference between the above two terms, i.e.,

$$\begin{aligned} D_t &= C_t - (S_t - K)^+ \\ &= \begin{cases} C_t & S_t < K \\ C_t - S_t + K & S_t \geq K \end{cases} \\ &= \begin{cases} S_t \mathcal{N}(d_1) - Ke^{-r(T-t)} \mathcal{N}(d_2) & S_t < K \\ S_t (\mathcal{N}(d_1) - 1) + K\left(1 - e^{-r(T-t)} \mathcal{N}(d_2)\right) & S_t \geq K \end{cases} \end{aligned}$$

Bear in mind that if we check into the delta of the D_t, you can easily find out that

$$\delta_t = \begin{cases} \mathcal{N}(d_1) \geq 0 & S_t < K \\ \mathcal{N}(d_1) - 1 \leq 0 & S_t \geq K \end{cases}$$

thus the maximum value is binding at $S_t = K$, with the objective value

$$S_t \mathcal{N}\left(\frac{\left(r + \frac{\sigma^2}{2}\right)\sqrt{T-t}}{\sigma}\right) - Ke^{-r(T-t)} \mathcal{N}\left(\frac{\left(r - \frac{\sigma^2}{2}\right)\sqrt{T-t}}{\sigma}\right).$$

When $S_t \to \infty$, the price asymptotically approaches $K\left(1 - e^{-r(T-t)}\right)$ since $\lim_{S_t \to \infty} \left(S_t\left(\mathcal{N}(d_1) - 1\right)\right) = 0$ and $\mathcal{N}(d_2) \to 1$.

Date: June 10, 2012.